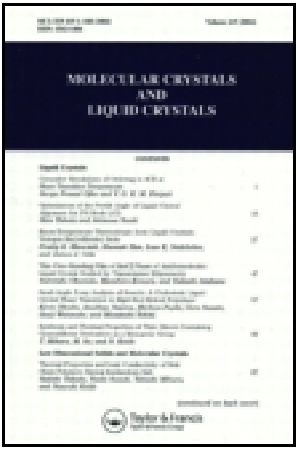

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Effect of Material Nonlinearity on the Acceleration Twist Waves in Liquid Crystals

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Effect of Material Nonlinearity on the Acceleration Twist Waves in Liquid Crystals

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Employing the Ericksen-Leslie continuum theory of liquid crystals we present expressions for the speed and the amplitude of acceleration and higher-order twist waves propagating in nematic and cholesteric liquid crystals. Such waves were first discussed by Fergason and Brown and also by Ericksen.

We show that it is due to nonlinearity of the stored energy density that acceleration twist waves can survive for all time in liquid crystals and possibly lead to phase transitions associated with shock waves.

INTRODUCTION

Recently, Shahinpoor[1] has analysed the propagations of finite twist waves in liquid crystals. He has obtained explicit relations for the speeds and the amplitudes of acceleration and higher-order (weak) twist waves in nematic and cholesteric liquid crystals employing a continuum theory due to Ericksen and Leslie.

In the present work we present a discussion on the effect of material nonlinearity on the ultimate survival of accelerations twist waves in liquid crystals. Shahinpoor[1] in his analysis showed that material nonlinearity has no effect on weak twist waves which are all damped. However, acceleration twist waves are very much sensitive to a coefficient $\hat{E}_t$, hereafter called the

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second twist tangent modulus, given explicitly by

$$\hat{E}_t \equiv \left.\frac{\partial^3 \hat{W}}{\partial \theta_{,X}^3}\right|_{X=Y(t)}, \quad W = \hat{W}(\theta_{,X}, X), \tag{1}$$

where W is the stored twist energy potential, $\theta(X, t)$ is the twist angle, and X is a material coordinate associated with the direction of propagation of twist waves, whose wave front is at $X = Y(t)$ at time t.

GOVERNING EQUATIONS

In order to be brief we only reproduce the results obtained in Ref. 1 and then proceed with the present analysis.

The intrinsic velocity U of every twist wave of order $N \geq 2$, (Theorem 4.1 of Ref. 1), satisfies

$$U^2(t) = \rho_1^{-1} E_t, \; \rho_1 > 0, \tag{2}$$

where E_t is called the instantaneous tangent modulus evaluated at $Y(t)$ and is given by

$$E_t = \left.\frac{\partial^2 \hat{W}}{\partial \theta_{,X}^2}\right|_{X=Y(t)} = \left.\frac{\partial T}{\partial \theta_{,X}}\right|_{X=Y(t)}, \tag{3}$$

where $T(\theta_{,X}, X)$ is the twist couple, and ρ_1 is the micro-inertia of a single macromolecule. A necessary and sufficient condition for the reality of speeds of twist waves of order $N \geq 2$ is then

$$E_t \geq 0. \tag{4}$$

This essentially implies that the slopes of the couple-twist gradient $(T - \theta_{,X})$ curves are positive definites.

For simplicity we consider twist waves propagating into a homogeneous liquid crystal medium initially at rest. This essentially implies that just before the wave propagations the medium resembles either a uniformly aligned nematic phase or a linearly twisted cholesteric phase at rest. Thus the amplitudes of acceleration and weak twist waves are then given, respectively, by:

$$\frac{a(t)}{a(0)} = \frac{[E_t(t)/E_t(0)]^{1/4} \exp(\frac{1}{2}\lambda_1 \rho_1^{-1} t)}{1 + a(0) \int_0^t [E_t(t)/E_t(0)]^{1/4} E_t^{-1} U^{-1} \hat{E}_t \exp(\frac{1}{2}\lambda_1 \rho_1^{-1} \tau) d\tau}, \tag{5}$$

For $N = 2$

$$\frac{a(t)}{a(0)} = [E_t(t)/E_t(0)]^{1/4} \exp(\tfrac{1}{2}\lambda_1 \rho_1^{-1} t), \quad \text{for } N \geq 3, \tag{6}$$

where $\lambda_1 < 0$ is a material constant. It is a common prejudice to consider ρ_1 as extremely small even for large macromolecules, and $[E_t(t)/E_t(0)]$ as unity or slowly varying bounded function. Thus, jerk twist waves, i.e., $a(t) = [\![\dddot{\theta}]\!]$, and higher order twist waves are normally damped in homogeneous liquid crystals which are initially at rest. However, acceleration twist waves behave differently inasmuch as the time behavior of the following expression is concerned:

$$I(t) = \int_0^t [E_\tau(\tau)/E_\tau(0)]^{1/4} E_\tau^{-1} U^{-1} \hat{E}_\tau \exp(\tfrac{1}{2}\lambda_1 \rho_1^{-1}\tau) d\tau. \tag{7}$$

Because of the smoothness of the functions appearing in the integrand in (7), $I(t)$ is strictly monotone, as it starts to vary from the initial time $t = 0$ for which $I(0) = 0$. Thus, a necessary condition for the existence of a finite limit $a(t)$ of an acceleration twist wave is that

$$1 + a(0)I(t) > 0. \tag{8}$$

Note that if $\hat{E}_t = 0 \; \forall \; t \in [0, t)$, that is if the material has a linear couple-twist gradient curve[2] (Figure 1, curve (I)), i.e., the material is linear, then acceleration twist waves behave the same as higher order twist waves in time and they normally damp out. However, if $\hat{E}_t \neq 0$ for $t \in [0, t)$ then the acceleration twist waves may lead to either finite or infinite limits. The latter would correspond to the evolution of twist shock waves which are normally believed to create phase transitions.[3]

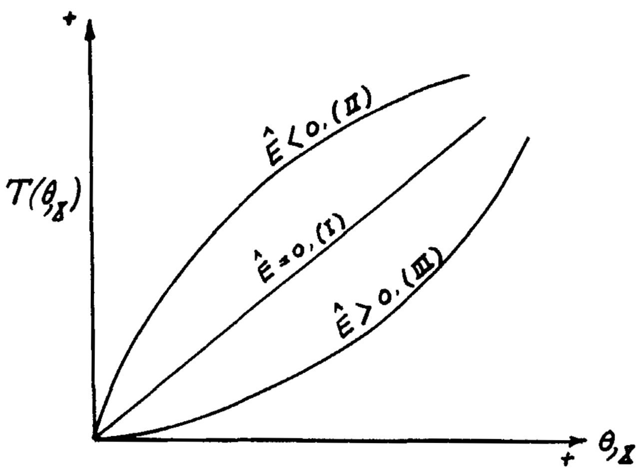

FIGURE 1 Typical $T - \theta_{,x}$ curve for twisting of liquid crystals

It is clear from the expression (5) and the strict monotonicity of $I(t)$ that a necessary condition for the evolution of twist shock waves is that

$$a(0)I(t) < 0, \quad \text{(for } t \text{ small)}. \tag{9}$$

Without loss of generality we can assume that $U > 0$ and thus

i) If $a(0) < 0$, $\hat{E}_t > 0$, $t \in [0, t)$ then a twist shock wave is likely to evolve at a $t^* \geq t$. This means for materials of class (III) in Figure 1, an expansion twist acceleration wave can create a twist shock wave.

ii) If $a(0) > 0$, $\hat{E}_t < 0$, $t \in [0, t)$, then a twist shock wave is likely to evolve at a $t^* \geq t$. This means for materials of class (II) in Figure 1, compressive twist acceleration wave can create a twist shock wave.

It is also interesting to note that for certain other materials acceleration twist waves may survive for all time with finite amplitude. This rather involves a large class of materials for which we choose to consider a smaller class. Consider liquid crystals for which

$$\hat{E}_t = \alpha^* E_t^{5/4}, \tag{10}$$

where α^* is an arbitrary constant. These liquid crystals belong to a larger class B^*, which is a subclass of B, such that the twist energy potential is represented by

$$W(\theta_{,X}) = 256\,\frac{[\alpha^*\theta_{,X} + \beta^*]^{-2}}{6\alpha^{*2}} + \gamma^*\theta_{,X}, \tag{11}$$

where

$$\pm\,\tfrac{1}{4}\beta^* = \left\{\left.\frac{\partial^2 W}{\partial\theta_{,X}^2}\right|_{\theta_{,X}=0}\right\}^{-1/4} = E(0)^{-1/4},\ \gamma^* = T(0) + \frac{256}{3\alpha^*}\,\beta^{*-3}. \tag{12}$$

The couple-twist gradient $(T - \theta_{,X})$ curve is given by the following relation

$$T(\theta_{,X}) - T(0) = \frac{-[\alpha^*\theta_{,X} + \beta^*]^{-3} + \beta^{*-3}}{(3\alpha^*/256)}, \tag{13}$$

and obviously condition (4) is satisfied, namely that

$$E_t = \left.\frac{\partial^2 W}{\partial\theta_{,X}^2}\right|_{X=Y(t)} = 256[\alpha^*[\theta_{,X}]_t + \beta^*]^{-4} > 0. \tag{14}$$

For this case expression (5) reduces to

$$\frac{a(t)}{a(0)} = \frac{[E_t(t)/E_t(0)]^{1/4}\exp(\frac{1}{2}\lambda_1\rho_1^{-1}t)}{1 + 2a(0)\alpha^*\lambda_1^{-1}\rho_1^{3/2}E_t^{-1/4}(0)[\exp(\frac{1}{2}\lambda_1\rho_1^{-1}t) - 1]}. \tag{15}$$

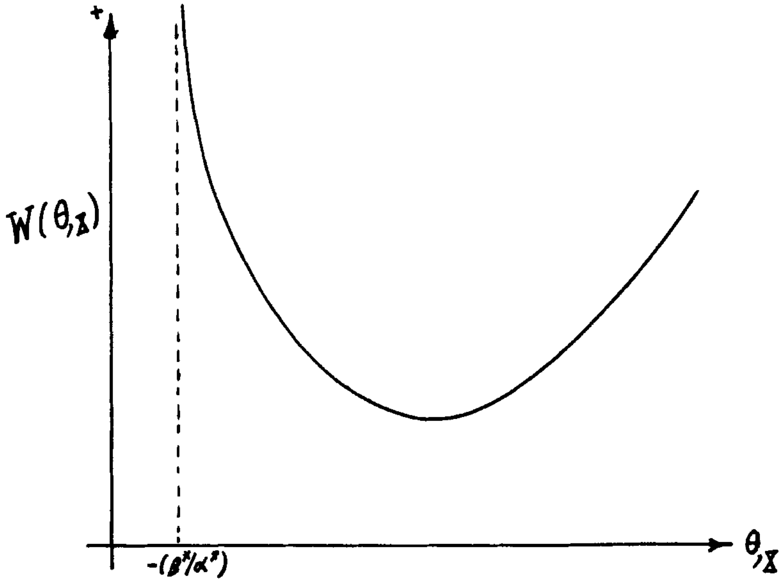

FIGURE 2 A typical $W - \theta_{,X}$ curve for cholesteric liquid crystals

Note that now by appropriate choice of the coefficients $a(0)$, α^*, λ_1, ρ_1 and $E_t(0)$, we can eliminate the damping effect completely and force acceleration waves to propagate as desired. For example, if

$$2a(0)\alpha^*\lambda_1^{-1}\rho_1^{3/2}E_t^{-1/4}(0) = 1, \tag{16}$$

then the acceleration twist waves propagate according to

$$\frac{a(t)}{a(0)} = [E_t(t)/E_t(0)]^{1/4}, \tag{17}$$

where $E_t(t)$ is given by (14).

From the symmetry restrictions on nematics it is clear that the representation (11) is not physically realizable for nematic liquid crystals. However, for cholesteric liquid crystals the representation (11) is physically sound and meaningful. A typical energy curve for this case possesses a minimum as depicted in Figure 2.

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